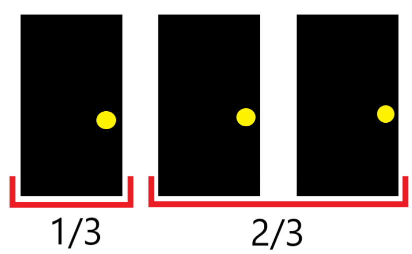
1 The game

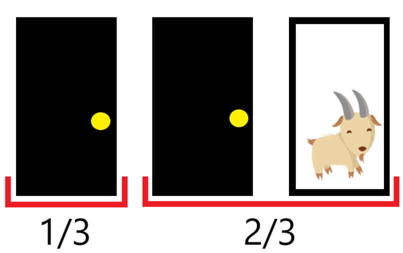
1.1 How to play

Although the game originally came from the American television game show Let's Make a Deal and being named after its host Mr Monty Hall. Originally participants in the show where shown 3 separate doors and asked to choose one with two of the doors having goats and one having the newest sports car; once a participant had chosen there door the host, Mr Monty Hall would open another door separate from the one that the participants had chosen and showed that there was a goat behind it. Meaning that the participants knew the sports car was behind one of the two last remaining doors. The host would then ask the participants if they wanted to switch. Most people choose not to or do on the assumption of a evenly split probability of the sports car behind the last door. But this is in fact wrong as we will see below.

1.2 The visual solution



We first need to understand that as soon as we choose a door the other two doors get bunched together and there probability is combined to $\frac{2}{3}$. Although this seems wrong and we would think of each door individually with a $\frac{1}{3}$ probability each they are not individual as just like in the game show we know one of the doors in the $\frac{2}{3}$ bundle has a goat behind it. And this will be conformed by one door being opened. However we keep the 2/3 probability value even though there can only be one door in that bunch the sports car could be behind.



As we see in the diagram and explained by the paragraph above the bundled $\frac{2}{3}$ probability for the two doors is actually $\frac{2}{3}$ for one door if we want to find the sports car as we know one of them is a goat and not the sports car. Therefore it makes the most sense to switch the door as we have double the chance of getting the sports car if we switch as shown in the diagram. We can only group the other options together rather than the one we have chosen as we never get to open our door till the very end where as with the other two doors we know what one of them has behind it giving us a statistical advantage if we chose to switch.

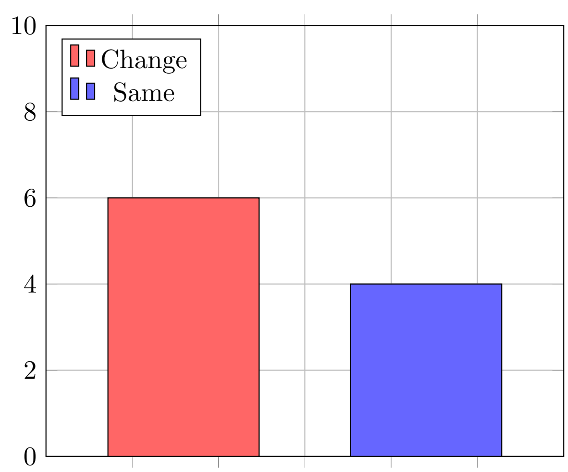
2 Computational Proof

2.1 Source Code

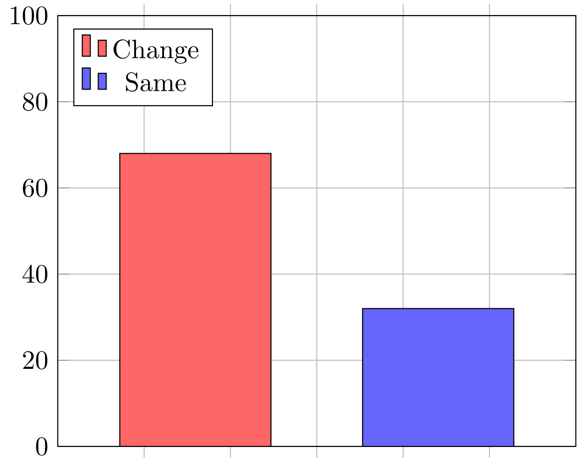
```
import random

tally = {"Change": 0, "Same": 0}
#Number of time we want to simulate the game
n = 1000
for x in range(n):
    doors = [0, 1, 2]
    UsrChoice, winner = random.randint(0,2), random.randint(0,2)

    #Determining which door we want to open
    if (winner+1)%3 == UsrChoice:
        doors.pop((winner+2)%2)
    else:
        doors.pop((winner+1)%2)

    #If the person doesen't change
    if winner == UsrChoice:
        tally["Same"] += 1
    #If they do change
    else:
        tally["Change"] += 1
print(tally)
```

2.2 Game Simulations

n = 10



n = 100



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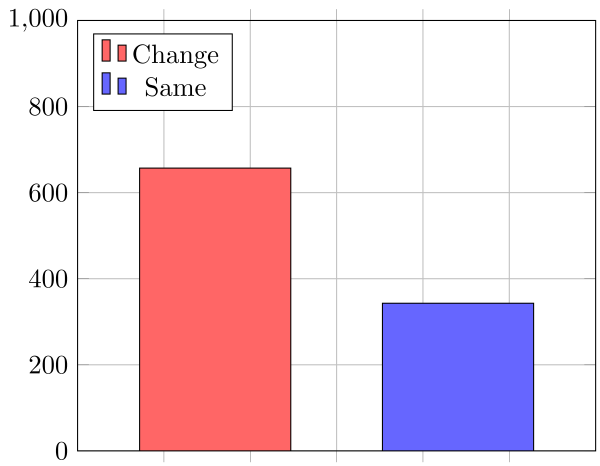
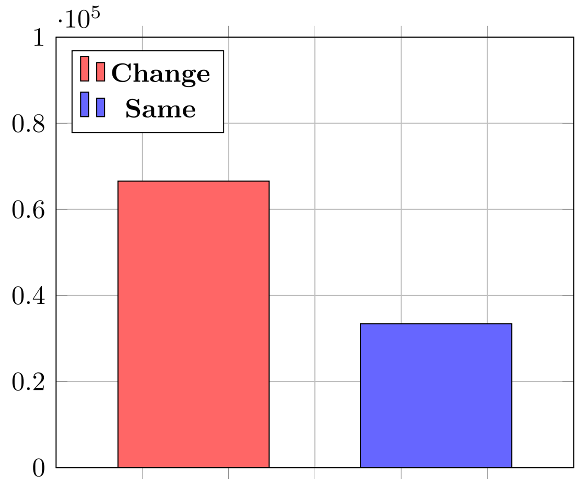
n = 1000

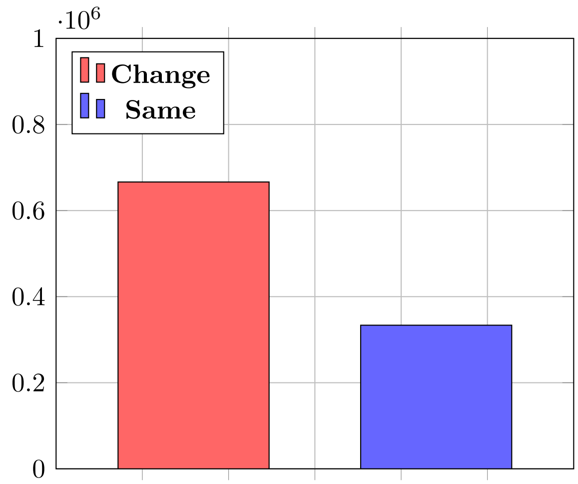


$n = 10^5$



$\cdot 10^5$
1
0.8
0.6
0.4
0.2
0
Change
Same

$n = 10^6$



$\cdot 10^6$
1
0.8
0.6
0.4
0.2
0
Change
Same

2.3 Raw Data

n	**Strategy**	**Success (%)**
10	Stay	40.0
10	Change	60.0
100	Stay	32.0
100	Change	68.0
1000	Stay	34.3
1000	Change	65.7
10^5	Stay	33.4
10^5	Change	66.6
10^6	Stay	33.3
10^6	Change	66.6

Towards the end of the table we see how the law of big numbers shows that as $n \to \infty$ we get to our estimated probability values for the game. That being that switching gives us a $\frac{2}{3}$ chance of winning while staying with the original door gives us a $\frac{1}{3}$ chance of winning over time. It should also be noted that this approach scales; if there where 100 doors instead of 3 the change of winning if you change doors turns to $\frac{99}{100}$ while if you stay on the same door you have a tiny $\frac{1}{100}$ chance.

3 Sources

3.1 Websites

web.mit.edu/rsi/www/2014/files/MiniSamples/MontyHall.Old.Bad/montymain.pdf
youtube.com/watch?v=4Lb-6rxZxx0
en.wikipedia.org/wiki/Monty_Hall_problemSimple_solutions
whitman.edu/documents/Academics/Mathematics/2019/Lopez-Martinez-Keef.pdf
digitalcommons.unl.edu/cgi/viewcontent.cgi?article=1019context=mathmidexppap
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sites.psu.edu/siowfa13/2013/12/04/probability-and-monty-hall-poblem/

3.2 Other

Lots of Latex code taken from other projects I have made in the past in order to save time. All data was gathered on the same machine using the source code shown.